



a review of Decomposition spaces, incidence algebras and Mobius inversion. II: Completeness, length filtration, and finiteness by Galvez-Carrillo, Imma; Kock, Joachim; Tonks, Andrew

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Decomposition spaces, incidence algebras and Möbius inversion. II: Completeness, length filtration, and finiteness. (English) [Zbl 1403.18016](#)

Adv. Math. 333, 1242-1292 (2018).

In the first part of this trilogy [Adv. Math. 331, 952–1015 (2018; [Zbl 1403.00023](#))] the authors have introduced the notion of decomposition space as a generalization for incidence coalgebras, which is equivalent to the notion of unital 2-Segal space of [T. Dyckerhoff and M. Kapranov, “Higher Segal spaces. I”, Preprint, [arXiv:1212.3563](#)]. The principal objective in this second part is to establish a Möbius inversion principle within the framework of *complete* decomposition spaces, and to analyze their associated finiteness conditions to ensure incidence coalgebras and Möbius inversion descent to classical level of \mathbb{Q} -vector spaces on taking the homotopy cardinality of the objects involved, which results in *Möbius decomposition spaces* as a generalization of Möbius categories in [P. Leroux, Cah. Topologie Géom. Différ. Catégoriques 16, 280–282 (1976; [Zbl 0364.18001](#))]. Möbius decomposition spaces cover more coalgebra constructions than Möbius categories, comprehending the Faà di Bruno and Connes-Kremer bialgebras.

Reviewer: [Hirokazu Nishimura \(Tsukuba\)](#)

MSC:

[18G30](#) Simplicial sets; simplicial objects in a category
[16T10](#) Bialgebras
[06A11](#) Algebraic aspects of posets
[05A19](#) Combinatorial identities, bijective combinatorics
[55U35](#) Abstract homotopy theory; axiomatic homotopy theory

Cited in **2** Reviews
 Cited in **3** Documents

Keywords:

decomposition space; 2-Segal space; incidence algebra; Möbius inversion; homotopy cardinality; length filtration

Full Text: [DOI](#)

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